Lectured by Dr. H.K. Yadav Date-18-05-2020
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B.Sc-part-I(H). paper-I. Algebra characteristic of ring.

Q.1 Define characteristic of a ring with examples.

Ans: Characteristic of a ring: →

Let R be a ring with zero element '0' and suppose there exists a positive integer n such that

$$n.a = a+a+a+ \cdots +a \text{ (upto n terms)} = 0$$

$\forall a \in R$. Then the smallest such positive integer n is called the characteristic of a ring.

If there does not exist such positive integer n such that $na = 0 \ \forall a \in R$ and $a \neq 0$, then the ring R is said to be of characteristic zero or infinite.

For example:

The ring of integers is of ~~characteristi~~ characteristic zero and the ring of rational number is also of characteristic zero.

If $I_6 = \{0,1,2,3,4,5\}$, then the ring $\{I_6, '+_6', 'x_6'\}$ is of characteristic 6, because $6a = 0. \ \forall a \in I_6$.

Q. 2. Prove that The characteristic of a ring with unity is zero or $n > 0$ according as the unity element 1 regarded as a member of the additive group of the ring has the order zero or n.

proof: Let R be a ring with unity element 1 and if order of 1 is zero, then it is obvious that the characteristic of the ring is zero. Let the order of 1 is finite say n, then

$$1+1+1+ \cdots +1 \text{ (n times)} = 0 \Rightarrow n.1 = 0. \quad \text{— (I)}$$

Let a be any non-zero element of R, then we have

$$na = a+a+a+ \cdots +a \text{ (n times)}$$
$$= 1.a + 1.a + 1.a + \cdots + 1.a \quad [\because R \text{ is with unity}]$$
$$= [1+1+1+ \cdots +1 \text{ (n times)}].a$$
$$= (n.1)a$$
$$= 0.a = 0 \quad [\text{by I}]$$

$\Rightarrow$ order of $a \leq n$

Hence, the characteristic of R is n.

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Q.3. Prove that the characteristic of an integral domain is zero or $n>0$ according as the order of any non-zero element of the integral domain regarded as a member of additive group of the integral domain either zero or n.

Proof: Let D be an integral domain and let a be any non-zero element of D.

If the order of a is zero, then obviously the characteristic of D is zero.

Let us suppose that the order of a is $n>0$, then we have,

$$na = 0 \text{ ——— (I)}$$

Furthermore, suppose b is any other non-zero element of D.

Multiplying both sides of (I) by b, we have

$$(na)b = 0.b$$

$$\Rightarrow [a+a+a+\cdots+a \text{ (n times)}]b = 0 \quad [\because 0.b = 0]$$

$$\Rightarrow ab+ab+ab+\cdots+ab \text{ (n times)} = 0$$

[By right distributive law]

$$\Rightarrow a[b+b+b+\cdots+b \text{ (n times)}] = 0$$

[by left distributive law]

$$\Rightarrow n(nb) = 0$$

$$\Rightarrow nb = 0 \quad [\because a \neq 0] \text{ ——— II}$$

But $o(a) = n \Rightarrow n$ is the least positive integer such that $na = 0$. Also, $n.0 = 0$

Thus (I) and (II) implies that n is the least positive integer such that

$$nx = 0, \ \forall x \in D.$$

Hence, the characteristic of D is n.

Proved

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